The Informed Educator Series

Elements of Highly Effective Mathematics Programs

Issues concerning mathematics education run the gamut from discussions about the problems with finding highly qualified math teachers to questions concerning skills students will need when they enter the workforce. Even more basic, a wide range of parties—educators, parents, business leaders, policymakers, etc.—are engaged in sometimes contentious discussions about what should be taught and how. The "environment" of math education has also increased in complexity, with states mandating standards and assessments that provide more explicit direction for local schools and districts.

However, there is an issue about which there is no disagreement. School leaders and teachers across the country have the responsibility for assuring that their students achieve math standards and are ready to move on to higher education or employment with the skills they need to succeed. The task is complex, requiring information about good math instruction to support efforts at the district, school, and classroom levels.

This *Informed Educator* presents research-based information about good math instruction, discussing what gets taught and how it should be taught. Areas highlighted include: skills- vs. inquiry-based instruction, supporting struggling students, the role of school and district leaders, and staff development for teachers and principals.

What Gets Taught

When considering content knowledge and skills, it is obvious that schools need to look first at state standards students are expected to master. Over the past decade, these standards have become more specific "driven, in part, by increased accountability in the form of state-mandated testing and, not coincidentally, by a call from teachers asking for more guidance in what mathematics to focus on at particular grades" (Reys, Dingman, Sutter, and Teuscher 2005, 2).

Those outside education might assume that what and when particular mathematical concepts are presented are fairly similar state to state—reflecting both some consistency of opinions about what students should learn as well as an understanding about the development of children's cognitive abilities. However, after analyzing information on current state standards, researchers Reys and Lappan found that:

> Mathematics learning expectations vary across the states along several dimensions, including level of specificity, language used to convey learning goals, and grade placement of specific learning expectations (2007, 677).

These researchers characterize the grade level placement of academic expectations as "dramatically different" (2007, 678) and provide some details:

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> Some state standards introduce computations with fractions (using common fractions such as 1/2) as early as grade 1, while others begin instruction on the topic in grade 3 or 4. Some state standards include an expectation for students to be fluent in computing in fractions by the end of grade 5; and other state standards include this expectation at grade 8 (2007, 678).

Research studies comparing math instruction in the U.S. and other countries have pointed to an underlying problem with many of our standards-based systems. Typically, these systems address many standards at each grade level—encouraging the development of a curriculum that has been characterized as a mile wide but an inch deep.

In a study for the U.S. Department of Education, Ginsburg, Leinwand, Anstrom, and Pollock compared mathematics curriculum, instruction, and textbooks used in seven states with similar elements in Singapore. Singapore was selected for the comparison because its students ranked first in the world in mathematics on the 2003 Trends in International Mathematics and Science Study, and it was hoped an analysis of its practices might provide some helpful lessons for U.S. educators. In the view of the authors of the study,

> Singapore's framework . . . lays out a balanced set of mathematical priorities centered on problem solving. It includes an emphasis on computational skills along with more conceptual and strategic thinking processes. The framework covers a relatively small number of topics in-depth and carefully sequenced grade-by-grade, following a spiral organization in which topics presented at one grade are covered in later grades, but only at a more advanced level. Students are expected to have mastered prior content, not repeat it (2005, xi.)

Three of the states studied used frameworks similar to those used in Singapore, and the researchers pointed to the relative success schools in these states have had with having students meet standards. In their view, focusing on a few highly-detailed curricular goals both provides important direction for teachers and allows instruction to focus on moving all students to mastery.

Goals of Effective Mathematics Instruction: Some Fundamental Principles

- Basic skills with numbers continue to be vitally important for a variety of everyday uses. They also provide crucial foundation for the higher-level mathematics essential for success in the workplace which must now also be part of a basic education. Although there may have been a time when being to able to perform extensive paper-and-pencil computations mechanically was sufficient to function in the workplace, this is no longer true. Consequently, today's students need proficiency with computational procedures. Proficiency, as we use the term, includes both computational fluency and understanding of the underlying mathematical ideas and principles.
- Mathematics requires careful reasoning about precisely defined objects and concepts. Mathematics is communicated by means of a powerful language whose vocabulary must be learned. The ability to reason about and justify mathematical statements is fundamental, as is the ability to use terms and notation with appropriate degrees of precision. By precision, we mean the use of terms and symbols, consistent with mathematical definitions, in ways appropriate for students at particular grade levels. We do not mean formality for formality's sake.
- Students must be able to formulate and solve problems. Mathematical problem solving includes being able to (a) develop a clear understanding of the problem that is being posed, (b) translate the problem from everyday language into a precise mathematical question, (c) choose and use appropriate methods to answer the question, (d) interpret and evaluate the solution in terms of the original problem, and (e) understand that not all questions admit mathematical solutions and recognize problems that cannot be solved mathematically (Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid, and Schaar 2005, online).